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A STUDY OF THE EFFECT OF MOISTURE CONTENT UPON THE EXPANSION AND CONTRACTION OF PLAIN AND REINFORCED CONCRETE

BY

TORATA MATSUMOTO



BULLETIN No. 126

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A STUDY OF THE EFFECT OF MOISTURE CONTENT
UPON THE EXPANSION AND CONTRACTION OF
PLAIN AND REINFORCED CONCRETE*

I. INTRODUCTION

1. *Object of Investigation.*—While the properties of concrete have been investigated for many years, attention has largely been given to considerations of strength alone. Studies of the less important properties, however, are also needed to explain phenomena that are observed in reinforced concrete structures, and to give information on questions relating to the appearance and durability of the material.

Aside from the action of direct load, deformations are produced in concrete by changes in temperature and in moisture content. With reference to temperature changes in reinforced concrete, it is well known that, regardless of differences in the mixture, concrete has practically the same coefficient of expansion as steel, so that the two materials contract or expand together. Moisture content, on the other hand, has the undesirable property of affecting concrete alone. Concrete, like wood, clay, and some other materials, expands when it absorbs moisture and contracts when it is dried; steel has no such action. After the concrete is poured the steel remains unchanged with changes in moisture conditions, while concrete ordinarily shrinks a considerable amount. Aside from the stresses set up in steel and concrete by the shrinkage of the latter, the resulting formation of cracks large or small will produce a condition which may be favorable to the corrosion of the steel or the disintegration of the concrete after repeated changes from dry to wet condition.

*This bulletin contains some of the results of experiments made by T. Matsumoto, a graduate student in Theoretical and Applied Mechanics at the University of Illinois, in 1918. Owing to his years of experience as an engineer on harbor works on the Island of Formosa, he was particularly interested in the question of the durability of concrete exposed to sea air and to the conditions found in a tropical climate. It is felt, however, that the results obtained apply to our ordinary conditions and may have many practical applications. The paper has therefore been revised by members of the Department of Theoretical and Applied Mechanics and prepared in form for publication as a bulletin of the Engineering Experiment Station.

The tests which are described were made to investigate the amount of shrinkage which may be expected in a mortar or a concrete, the relation between the change of moisture content and the change of length of these materials, the difference in shrinkage of plain and reinforced concrete, and the internal stresses set up in the latter. For purposes of comparison with the results obtained with concrete, a few tests were made on the effect of the absorption of water by sandstone and limestone.

2. *Acknowledgments.*—The work reported in this bulletin was done in the Laboratory of Applied Mechanics of the University of Illinois. Acknowledgment is made to PROFESSOR ARTHUR N. TALBOT, PROFESSOR H. F. GONNERMAN, and others, for helpful suggestions and aid.

II. MATERIALS USED IN TEST SPECIMENS

3. *Materials.*—The materials used in making the concrete test pieces were all of good quality. Universal portland cement, which passed the specifications of the American Society for Testing Materials as to fineness, soundness, tensile strength, and other qualities, was used. The fine aggregate was a well graded sand from Attica, Indiana. An average mechanical analysis from ten samples of the material is given in Table 1. The specific gravity of the sand was 2.69. The weight per cu. ft. when tamped into the mold with a rod was 111 lb., from which the voids in the dry material were estimated to be 34 per cent.

The coarse aggregate was gravel from Attica, Indiana. It was all screened to a size between $\frac{1}{4}$ and $\frac{1}{2}$ in.; larger sizes were not used because of the small dimensions of some of the test pieces. The gravel consisted of well rounded pebbles having a specific gravity of 2.71; its weight per cu. ft. was about 97 lb., making the voids in the coarse aggregate about 42 per cent. The steel used in the reinforced concrete test pieces consisted of plain round mild steel bars. In none of the tests did the measured stress in the steel approach the elastic limit of the material.

TABLE 1
MECHANICAL ANALYSIS OF SAND

SIEVE NO.	SIEVE OPENING IN INCHES	PER CENT PASSING THE GIVEN SIEVE
3	0.263	98.6
4	0.185	95.3
8	0.092	71.4
14	0.046	42.3
28	0.0232	18.0
48	0.0116	7.1
100	0.0058	2.6

All concrete was hand mixed, and each test piece was made from a separate batch of material. Since the size of the test pieces and the conditions of their storage varied greatly in the different tests, these details will be noted in the descriptions of the several sets of tests. In the various mortars and concretes used, the proportions of the ingredients are given by volume.

4. *The Coefficient of Expansion of Mortar and Concrete.*—The coefficient of expansion of mortar and concrete was measured for use in making a temperature correction of measurements. It was obtained from perfectly dried specimens only, so that there was no error due to the change in length with a change in moisture content of a specimen. The measurement of length was made by the use of a Berry strain gage, and the temperature was measured by a thermometer which was so placed among the test pieces that radiation of heat did not affect the reading.

The results obtained indicate that the coefficient of expansion of mortar and concrete is practically the same for different proportions and ages, provided that the same materials are used in all mixtures. The average values of the coefficient of expansion of 1:1, 1:2, and 1:3 mortar, and 1:1:2, 1:2:4, and 1:3:6 concrete, varied from 0.0000050 to 0.0000058 for one degree Fahrenheit, with no particular variation with the age or with the richness of the mixture.

III. EFFECT OF MOISTURE CONTENT ON LENGTH OF SPECIMEN

5. *Cement Mortar and Plain Concrete.*—Thirty test pieces, 2 by 3 by 24 in. in size, containing 2 steel plugs which formed a 20-in. gage line, were made of 1:1, 1:2, and 1:3 mortar, and 1:1:2, 1:2:4, and 1:3:6 concrete, five test pieces for each mixture. One day after the test pieces were made the molds were taken off, and initial measurements of the length and weight were made. All test pieces were then stored in a damp room and covered with double sheets of burlap which were kept wet. The measurements of length were made with a Berry strain gage of 20-in. gage length, using as a standard a steel bar immersed in water of known temperature. All measurements were corrected for the effect of temperature change. Changes in weight were taken to mean gains or losses in moisture content.

After 60 days of damp storage all test pieces were stored in air in a room where the relative humidity varied from 40 to 60 per cent when the room was heated by steam. As soon as the steam heating was discontinued with the advance of the season the relative humidity of the room increased, and it reached 90 per cent on cloudy days; whereupon the mortar and concrete absorbed moisture from the air and began expanding instead of contracting. After being kept 90 days in the air of this room the specimens were dried for 14 days in an oven at a temperature of 150 deg. F., and then for 15 days at 200 deg. F. The amounts of change of moisture and length during the test are shown in Table 2 and are indicated in Figs. 1 and 2.

As may be seen from Fig. 1, 1:1 mortar at seventy days contracted 0.025 per cent, notwithstanding the fact that it contained more moisture than its initial amount. From this phenomenon it can be concluded that the water which goes into the concrete may exist in two different ways, either combined with cement in a chemical compound or remaining in a state which affects the volume of the concrete. It also appears that concrete in which the total moisture content is kept constant for some length of time is liable to contract slightly. This slight contraction was observed on several occasions during the test. It follows that the absolute amount of contraction of concrete

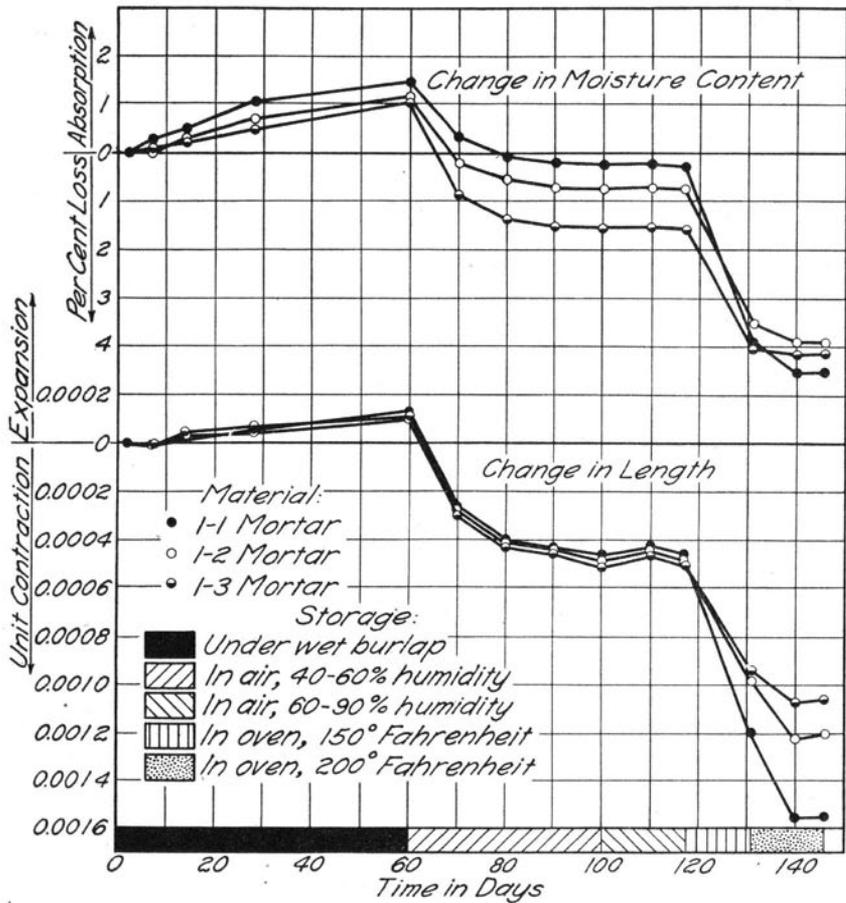


FIG. 1. CHANGE IN LENGTH WITH CHANGE IN MOISTURE CONTENT OF MORTAR

due to drying is indeterminate, because it depends upon the age of the concrete and the duration of the drying period. Figs. 1 and 2 show that in general these specimens of mortar and concrete expanded when they were kept wet. The richer mixtures absorb more water and consequently expand more than the leaner ones. Mortar and concrete lose moisture rapidly when placed in dry air and the loss is more rapid in lean mixtures than in rich ones. In air storage the shrinkage is greater for mortar than for concrete, and with artificial drying the shrinkage is greater for the richer mixtures.

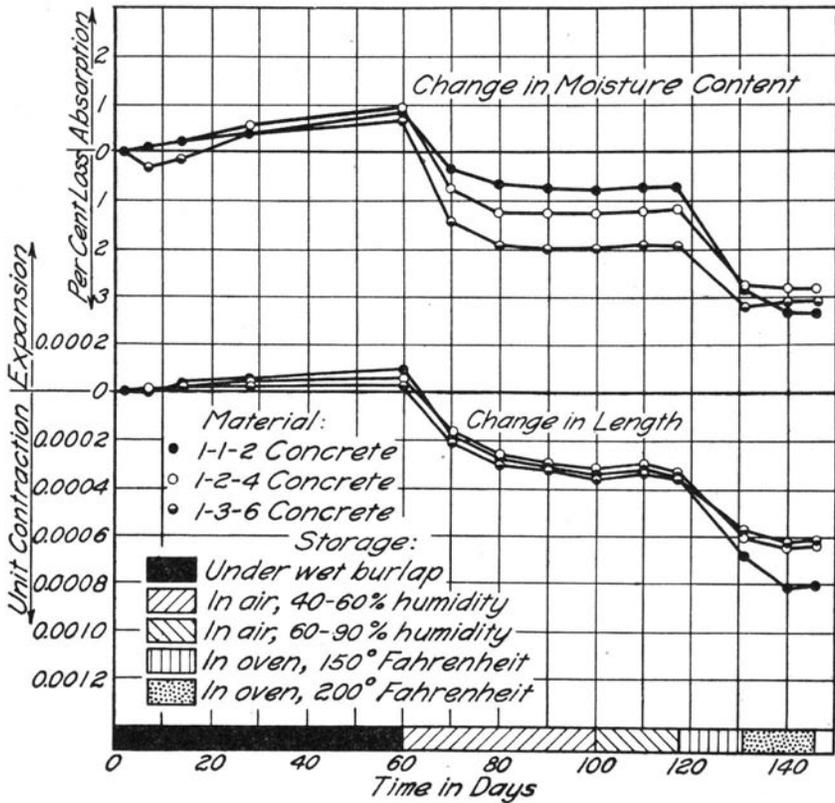


FIG. 2. CHANGE IN LENGTH WITH CHANGE IN MOISTURE CONTENT OF CONCRETE

For the specimens tested, the contraction of mortar and concrete at a temperature of 150 deg. F., which is somewhat higher than the highest outdoor temperature in summer in a tropical climate, was 0.12 per cent for 1:1 mortar, 0.10 per cent for 1:2 mortar, 0.09 per cent for 1:3 mortar, 0.07 per cent for 1:1:2 concrete, 0.06 per cent for 1:2:4 concrete, and 0.06 per cent for 1:3:6 concrete. The amount of expansion when specimens were kept wet reached 0.012 per cent in 1:1 mortar, and 0.002 per cent in 1:3:6 concrete, at the end of 60 days. Judging from these tests, shrinkage of concrete in a structure freely exposed to the air may be as much as 0.05 per cent in a 1:2:4

TABLE 2
 CHANGE IN MOISTURE CONTENT AND CORRESPONDING CHANGES IN LENGTH
 Expansion and Contraction are given in Per Cent of Gage Length

Pro- portion	Kept in Wet Condition 60 Days		Kept in Air of Room 58 Days		Dried in Oven at 150 Deg. F. 14 Days		Dried in Oven at 200 Deg. F. 15 Days	
	Gain of Moisture Per Cent	Expansion Per Cent	Loss of Moisture Per Cent	Contraction Per Cent	Loss of Moisture Per Cent	Contraction Per Cent	Loss of Moisture Per Cent	Contraction Per Cent
1:1	1.46	0.0124	0.26	0.0460	3.92	0.1204	4.54	0.1570
1:2	1.14	0.0100	0.71	0.0488	3.51	0.0991	3.97	0.1237
1:3	1.05	0.0101	1.57	0.0503	4.04	0.0944	4.18	0.1077
1:1:2	0.85	0.0091	0.68	0.0358	2.86	0.0673	3.33	0.0807
1:2:4	0.89	0.0055	1.25	0.0333	2.80	0.0605	2.82	0.0636
1:3:6	0.64	0.0019	1.91	0.0362	3.21	0.0576	3.17	0.0622

concrete.* In a structure such as a retaining wall or a road pavement, where one side of the structure is against a mass of earth, the shrinkage should be considerably less.

In a second set of tests twenty-four test pieces, having the same size and form as those in the tests previously described, were prepared from the same materials, half of mortar with the proportions of 1:2, and half of concrete with the proportions of 1:2:4. The test pieces

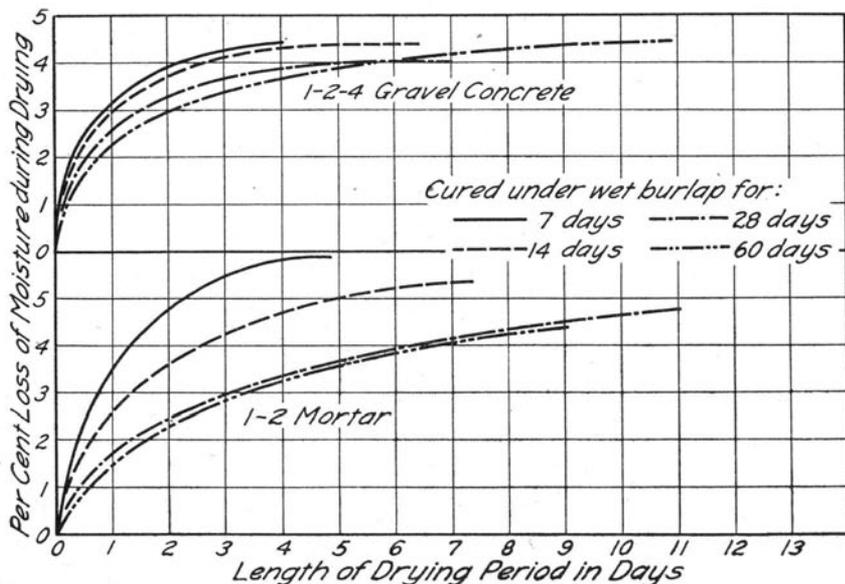
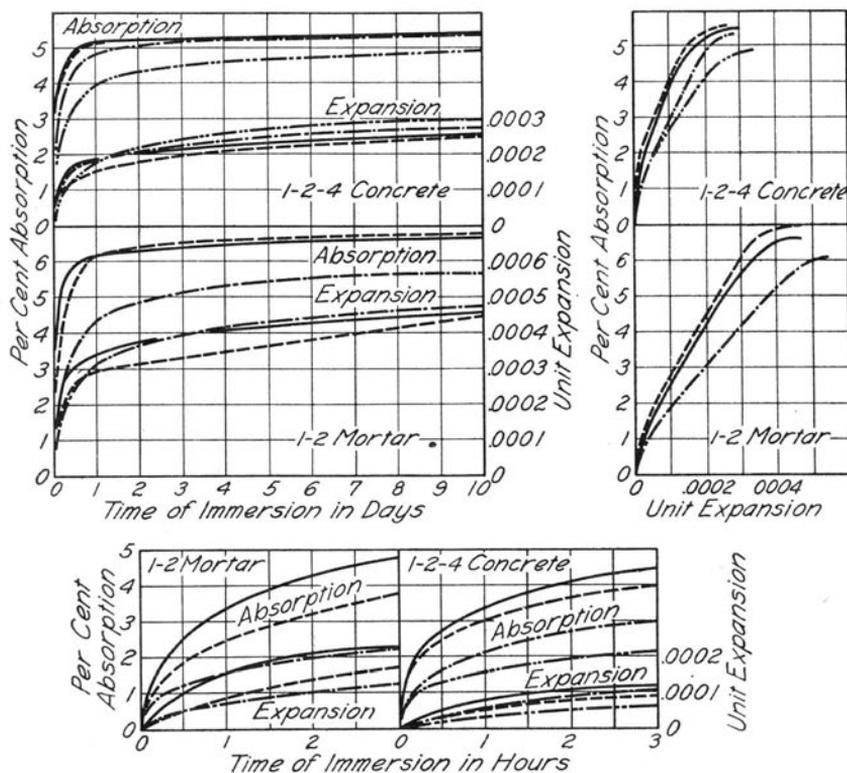


FIG. 3. LOSS OF MOISTURE FROM MORTAR AND CONCRETE WHEN DRIED AT 150 DEG. F. AFTER PRELIMINARY STORAGE NOTED IN DIAGRAM

* Quantitative information regarding the shrinkage of concrete over varying periods of time has been reported by a number of investigators, among whom are the following:

- McMillan, F. R. "Shrinkage and Time Effects in Reinforced Concrete." Studies in Engineering, No. 3, University of Minnesota, 1915.
- White, A. H. "Destruction of Cement, Mortars, and Concrete Through Expansion and Contraction." Proc. A. S. T. M., Vol. XI, 1911.
- Nichols, C. S. and McCullough, C. B. "The Determination of Internal Temperature Range in Concrete Arch Bridges." Engineering Experiment Station, Ames, Iowa, Bul. No. 30, 1913.
- Goldbeck, A. T. "The Expansion and Contraction of Concrete While Hardening." Proc. A. S. T. M., Vol. XI, 1911.
- Abrams, D. A. "Effect of Fineness of Cement." Structural Materials Research Laboratory, Lewis Institute, Chicago, Bul. No. 4, 1919.



Treatment of Specimens

- (1.) Cured under wet burlap for:
 - 7 days, - - - - 14 days, ———— 28 days, — · — · — 60 days
- (2.) Dried in oven at 150° F. as shown in Fig. 3.
- (3.) Immersed in water

FIG. 4. ABSORPTION AND CORRESPONDING EXPANSION OF MORTAR AND CONCRETE IMMERSSED IN WATER

were stored in a damp room and covered with burlap which was kept wet. Three specimens of each kind were taken out of the storeroom when they had hardened 7, 14, 28, and 60 days, respectively, in the damp condition, and were then dried in an oven at a temperature of 150 deg. F. until the loss of weight did not exceed 0.20 per cent during 24 hours. The test pieces were then immersed in clean water in a shallow tank. The lengths of the specimens were measured by a 20-in.

Berry strain gage and all measurements were corrected for the effect of temperature changes in the instrument. The results obtained are shown in Figs. 3 and 4, the loss in moisture being given in terms of weight of the mortar or concrete. The mortar weighed from 140 to 145 lb. per cu. ft., and the concrete weighed from 145 to 155 lb. per cu. ft.

From Fig. 3 it is seen that the rate of loss of moisture in mortar and concrete by drying decreases with the age of the specimen. In the mortar, the rate of loss in specimens 28 days and 60 days old, respectively, becomes nearly the same. The total loss in moisture in mortar does not seem to be greatly different from that in concrete at the four ages used.

A number of significant features of the second set of tests are brought out in Fig. 4:

(a) Dry mortar and concrete expand as soon as they absorb water.

(b) The rate of absorption upon immersion is much less for the older specimens during the first two or three days of immersion, although the total amount of absorption does not change very greatly for specimens of different ages.

(c) The rate of expansion due to immersion is also less for the older specimens during the first two or three days of immersion. For longer periods of time the amount of expansion is nearly the same for all ages of each material; but in every case the expansion is considerably greater for mortar than it is for concrete.

(d) The amount of the expansion of mortar and concrete is small until an absorption of nearly 2 per cent is reached. After this point the expansion becomes nearly proportional to the additional absorption of moisture. This holds true until a sudden decrease in the absorption occurs after about one day's immersion, beyond which point additional absorption gives a greatly increased expansion.

(e) The absorption of moisture by mortar or concrete is much more rapid than the drying out of moisture from the same material. As a result, the expansion due to wetting is more rapid than the contraction produced by drying. For example, a mortar 7 days old gained as much moisture in 2 hours' immersion as could be dried out by heating to 150 deg. F. for 24 hours.

Another experiment on the relation between absorption of moisture and expansion of concrete was made upon two specimens of 1:2:4 gravel concrete taken from a floor of the Western Newspaper Union Building in Chicago. The concrete was 9 years old. The sizes of the test pieces were 3.5 by 5 by 21 in., and 3.5 by 5 by 26 in. The specimens were dried for 13 days in an oven at a temperature of 150 deg. F. In drying, the average contraction of the concrete was 0.016 per cent for the two specimens. After immersion in water for 15 days the average absorption was 5.09 per cent and the resulting expansion 0.042 per cent. The results of the test are shown in Fig. 5.

6. *Sandstone and Limestone.*—For comparison with the behavior of mortar and concrete, a few tests were made on the absorption of water by stone. The specimens used were sandstone and limestone which had nearly the same porosity as the mortar and concrete tested. If the expansion of such material is due to the separation of particles, the same amount of expansion might be expected with stones of the same porosity.

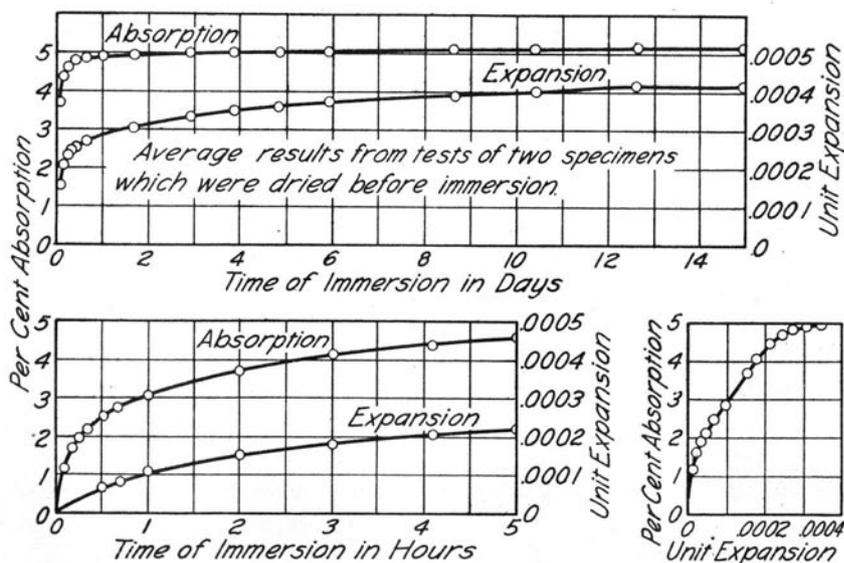


FIG. 5. ABSORPTION AND CORRESPONDING EXPANSION OF NINE-YEAR-OLD CONCRETE IMMERSSED IN WATER

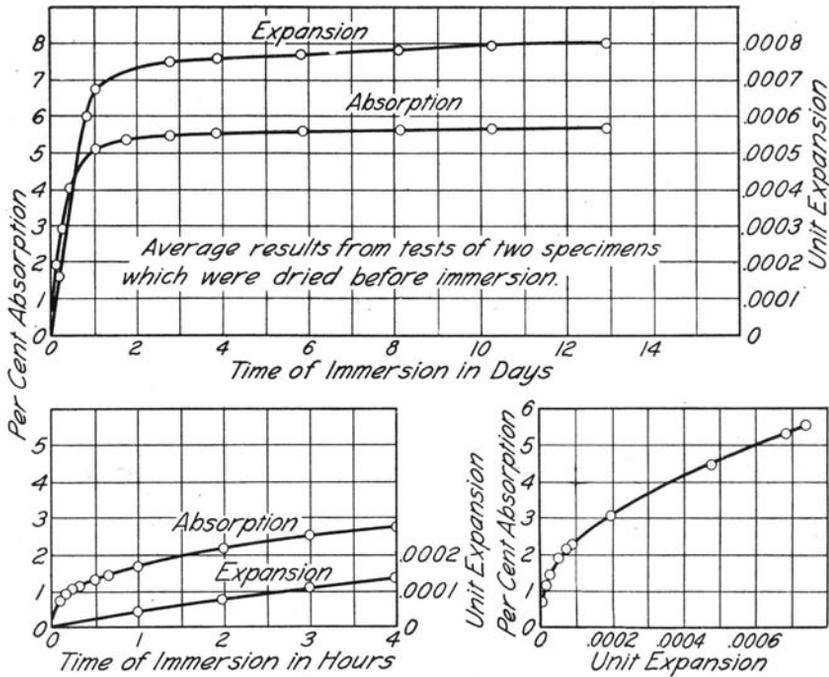


FIG. 6. ABSORPTION AND CORRESPONDING EXPANSION OF LAKE SUPERIOR RED SANDSTONE IMMERSIED IN WATER

Two specimens of a reddish brown sandstone of good quality, obtained from the Lake Superior region for use in the construction of a University building, were used in the test. The sizes of the specimens were 3 by 4 by 24 in. and 3.5 by 4 by 25 in. Both pieces were dried for 6 days in an oven at a temperature of 150 deg. F. After immersion in water for 13 days, the average absorption for the two specimens was 5.75 per cent, and the corresponding expansion was 0.080 per cent. By referring to Figs. 4 and 5 this amount of expansion is seen to be much greater than that of mortar or concrete after the same period of immersion. The results of the test are shown in Fig. 6.

The limestone specimens used in the test were a good quality of oölitic limestone of the kind used for building purposes, commonly known as Bedford Indiana stone. The stone has a porous structure. The two test pieces were 3.5 by 3.7 by 11.1 in., and 3.5 by 4.75 by 12.5

in. in size. The first specimen was dried for 7 days in an oven at 150 deg. F., with consequent loss of moisture, due to the drying, of 1.04 per cent, and no measurable contraction. The second specimen was dried for 11 days in an oven at 150 deg. F., with consequent loss of moisture of 1.42 per cent and no measurable contraction. They were then immersed in water. Fig. 7 shows that after an immersion of 6 days the average absorption was 6.1 per cent with practically no expansion.

The very rapid absorption and the lack of expansion of the limestone are in decided contrast to what occurred with all the other materials used. The behavior of the limestone throws doubt upon the theory that moisture entering a porous substance causes an expansion of material through separation of the particles.

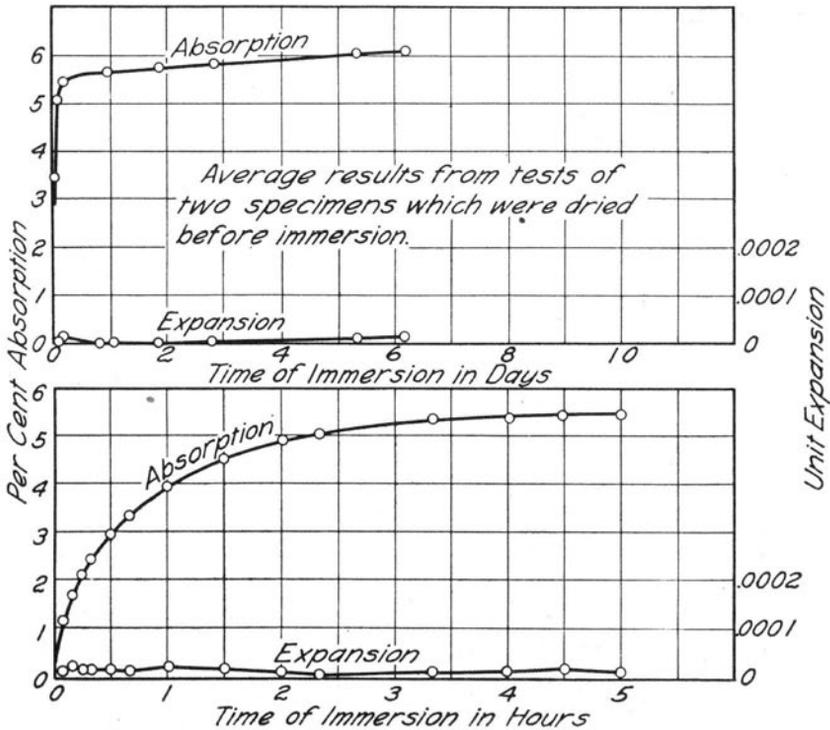


FIG. 7. ABSORPTION AND CORRESPONDING EXPANSION OF INDIANA OÖLITIC LIMESTONE IMMERSSED IN WATER

IV. SHRINKAGE STRESSES IN REINFORCED CONCRETE

7. *Analysis of Shrinkage Stresses.*—In the preceding tests on plain concrete, a considerable amount of information was obtained regarding the changes in length corresponding to changes in moisture content. It is desirable to know how such information can be applied to the action of reinforced concrete specimens. It is known that there is a well defined difference between the shrinkage in a set of plain concrete test pieces, and that in a set of reinforced concrete test pieces made from the same materials. This difference may be taken as due to the resistance of the reinforcement to the shrinkage of the concrete, which thus produces compressive stress in the steel and tensile stress in the concrete. It has been found possible to estimate the actual shrinkage stresses, both in steel and concrete, by comparison with the shrinkage of plain concrete under similar conditions.

When a plain concrete prism is dried out in air all particles have a tendency to move toward the middle of the prism, thus decreasing its length without causing cracks unless external forces resist this tendency. Consider one half of the length of a reinforced concrete prism as shown in Fig. 8. If there was no bond resistance between the concrete and the reinforcing bar, when shrinkage occurred the relative

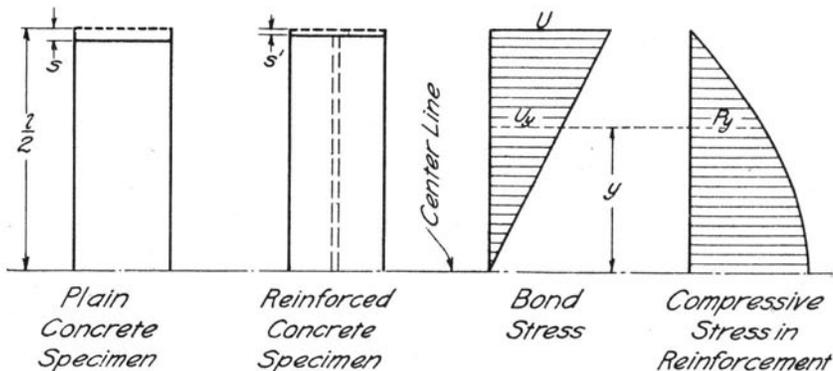


FIG. 8. DISTRIBUTION OF SHRINKAGE STRESSES IN REINFORCED CONCRETE SPECIMEN

displacement of concrete particles with reference to the reinforcement would be maximum at the end of the prism and zero at the center. Since resistance to this displacement actually is offered by the bond resistance between the concrete and the bar, the bond stress developed should be proportional to the amount of the tendency toward displacement. This bond stress is, therefore, greatest at the end of the bar and smallest at the center. If there is no slipping of the bar, a fairly reasonable assumption is that the bond stress varies uniformly from a maximum at the end of the bar to zero at the middle of the length of the bar.

The following notation will be used :

e = shrinkage in a unit length of plain concrete ;

$s = \frac{el}{2}$ total shrinkage in a half-length of a plain concrete prism ;

s' = total shrinkage in a half-length of a reinforced concrete prism ;

l = length of the prism ;

$a = pA$ = cross-sectional area of the reinforcing bar ;

A = cross-sectional area of the prism ;

o = circumference of the bar ;

u = maximum bond stress at the end of the bar ;

P_v = total compressive stress in bar = total tensile stress in concrete, at any section y distant from the center ;

$E_s = nE_c$ = modulus of elasticity of steel ;

E_c = modulus of elasticity of concrete.

Since the total compressive stress P_v , produced at any section, must equal the summation of the bond stresses between the section and the end of the bar, it follows that the maximum value of P_v occurs at the middle of the bar and is equal to $\frac{uol}{4}$. At any other point the stress P_v is given by the expression

$$P_v = \frac{uol}{4} \left[1 - \left(\frac{2y}{l} \right)^2 \right] \quad \dots \dots \dots (1)$$

This is the equation of a parabola with its vertex at the center of the bar as shown in Fig. 8.

The total deformation of the half-length of the bar is

$$s' = \int_0^{\frac{l}{2}} \frac{P_y dy}{aE_s} = \frac{uol^2}{12aE_s} \dots \dots \dots (2)$$

Since the total stresses in steel and concrete are equal in amount at any section, the total deformation in the concrete is

$$s - s' = \frac{el}{2} - s' = \int_0^{\frac{l}{2}} \frac{P_y dy}{AE_c(1-p)} = \left[\frac{uol^2}{12 AE_c(1-p)} \right] \dots (3)$$

From equations (2) and (3), disregarding the factor (1-p), the maximum bond stress is found to be

$$u = \frac{6 eaE_s}{ol(1+np)} \dots \dots \dots (4)$$

The total stress $P = \frac{1.5 eaE_s}{1+np}$

whence the maximum compressive unit stress $f_s = \frac{1.5 cE_s}{1 + np} \dots (5)$

and the maximum tensile unit stress

$$f_c = \frac{af_s}{A} = pf_s \dots \dots \dots (6)$$

The shrinkage stresses in the steel and concrete are thus seen to be directly proportional to the amount of shrinkage in plain concrete. It is seen that the stress in the steel decreases and the stress in the concrete increases with an increase in the percentage of reinforcement.

It must be remembered that equations (5) and (6) give maximum stresses; the average stresses over the length of the specimen are two-thirds as great. In the following experiments gage lengths were used which were five-sixths and five-twelfths of the length of the specimen; in these cases the average measured stresses should be 77 per cent and 94 per cent, respectively, of the maximum stress at the middle of the bar.

8. *Measurement of Shrinkage Stresses in Reinforced Concrete Specimens.*—Two sets of tests were made for the purpose of measuring

the shrinkage stresses in reinforced concrete specimens. For the first set of experiments six test pieces, 2 by 3 by 24 in. in size, were prepared, two of them being of plain concrete, two having two $\frac{3}{8}$ -in. round steel bars ($p = 3.68$ per cent), and two having two $\frac{1}{2}$ -in. round steel bars ($p = 6.54$ per cent). The proportions of the concrete were 1:2:4. The specimens were dried in the air of a room for 22 days; then they

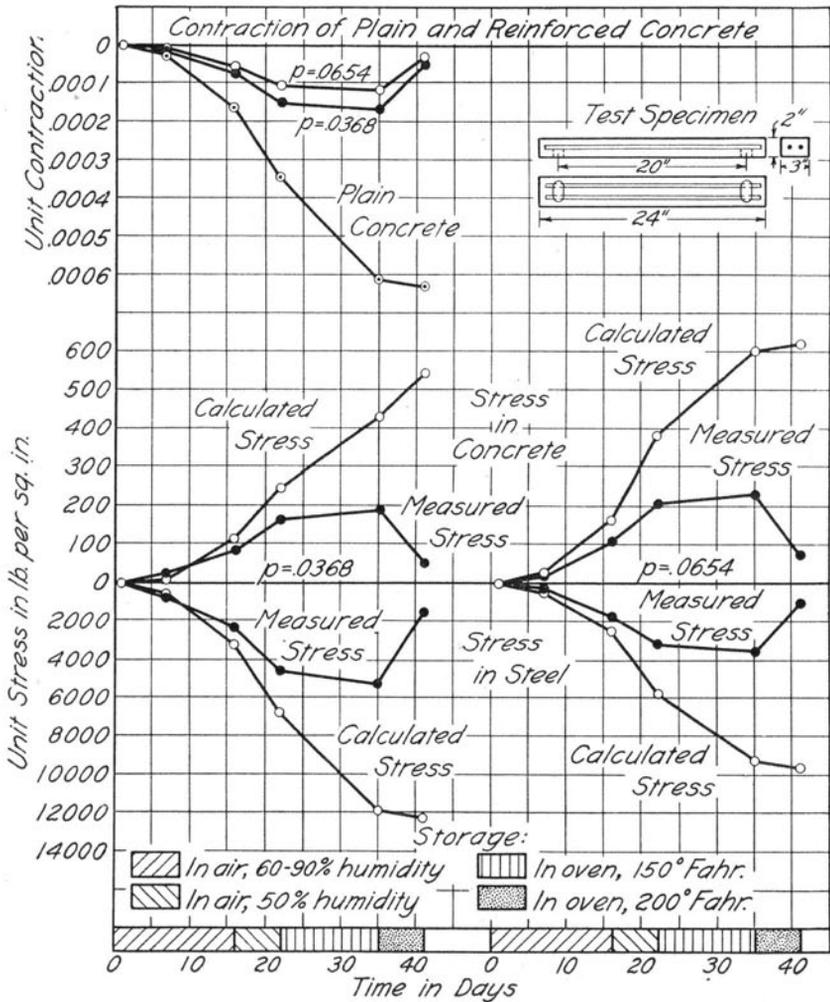


FIG. 9. SHRINKAGE STRESSES IN REINFORCED CONCRETE SPECIMENS

were dried in an oven for 13 days at a temperature of 150 deg. F.; and finally they were dried for 6 days at a temperature of 200 deg. F.

The change of length was measured on a 20-in. gage line between points on each embedded steel bar in the reinforced concrete test pieces, and between points on steel plugs embedded in the plain concrete test pieces. The results, which were very uniform in companion specimens, are shown in Fig. 9. The measured stresses in the steel were obtained by multiplying the measured deformation in the steel by the modulus of elasticity of steel; the curves marked "measured stress in concrete" were obtained by multiplying the measured stress in the steel by the percentage of reinforcement, see equation (6), and represent the average stress over the cross-section of the concrete. The calculated stress is the average calculated stress over the 20-in. gage length, or 77 per cent of the maximum stress given by equations (5) and (6).

As may be seen from Fig. 9, even reinforced concrete having a very high percentage of reinforcement contracts rapidly in air. When the specimens were 22 days old the shrinkage stress in the concrete reached 166 lb. per sq. in. in the specimens having 3.68 per cent reinforcement, and 204 lb. per sq. in. in the specimens having 6.54 per cent reinforcement.

The same specimens, after being dried 13 days longer at a temperature of 150 deg. F., developed stresses of 190 lb. per sq. in. and 240 lb. per sq. in., respectively, which nearly correspond to the ultimate tensile strength of the concrete. Furthermore, being dried at a temperature of 200 deg. F., the concrete failed in tension and showed two transverse cracks at points about 8 in. from each end.

This last case, however, may not be representative, because there may have been some injury to the texture of the concrete when it was heated beyond 200 deg. F. But the fact that the shrinkage stress in the concrete reached 170 to 200 lb. per sq. in. in 22 days, in the air of a room, may be sufficient reason for concluding that cracks will eventually be formed in concrete which has a high percentage of reinforcement.

The sudden drop of the shrinkage stress in the concrete when heated beyond 200 deg. F., as shown in Fig. 9, indicates injury to the concrete at this temperature; so that the use of reinforced concrete exposed to a temperature higher than 200 deg. F. may be dangerous unless proper precautions are taken.

A second set of tests was made in which smaller percentages of reinforcement were used. The size of the test pieces was 6 by 6 by 24 in. and they were made of 1:2:4 concrete. Deformations were measured on a gage length of 10 in. One piece was made without reinforcement, one with four $\frac{1}{4}$ -in., one with four $\frac{3}{8}$ -in., and one with four $\frac{1}{2}$ -in. round steel bars. A similar set of test pieces was made with the same amount of reinforcement, but which had in addition three anchor-lugs welded to each end of the reinforcing bars to guard against a slipping of the reinforcement during the setting of the concrete; a test piece of plain concrete was also included in this set. The size and form of the test pieces are shown in Figs. 10 and 11.

These specimens were left in the air of the room for 99 days. They were then dried in an oven at a temperature of 200 deg. F. The contraction which took place during the drying and the measured shrinkage stresses in both steel and concrete are shown in Figs. 10, 11, and 12. The measured stresses in steel and concrete were determined as in the preceding tests. The calculated stress is the average calculated stress over the 10-in. gage length, or 94 per cent of the stress given by equations (5) and (6).

In the case of the plain concrete, the results of the two sets of tests were quite uniform, both as regards the loss of moisture and the resulting contraction during setting and hardening. The difference between the contraction of plain concrete specimens and those having different amounts of reinforcement was quite regular at all conditions of storage, and is very instructive. The deformation in the reinforcing bars provided with lugs was greater than the deformation in the plain bar. This indicates that slipping of the bars took place with the latter during the shrinking of the concrete; the shrinkage stress is seen to be greater in reinforced concrete having good anchorage of the reinforcement. In general some slipping must be expected.

The measured stress in the steel reached 18 000 lb. per sq. in. in the specimen having reinforcement of 0.55 per cent, and the stress in the concrete reached 250 lb. per sq. in. in the specimen having reinforcement of 2.18 per cent. This stress in the steel exceeds the accepted working stress of soft steel and the stress in the concrete is nearly equal to the ultimate tensile strength of concrete of this kind.

When these specimens were heated above 200 deg. F. these stresses both in concrete and steel suddenly dropped down, probably due to injury of the concrete by the heat.

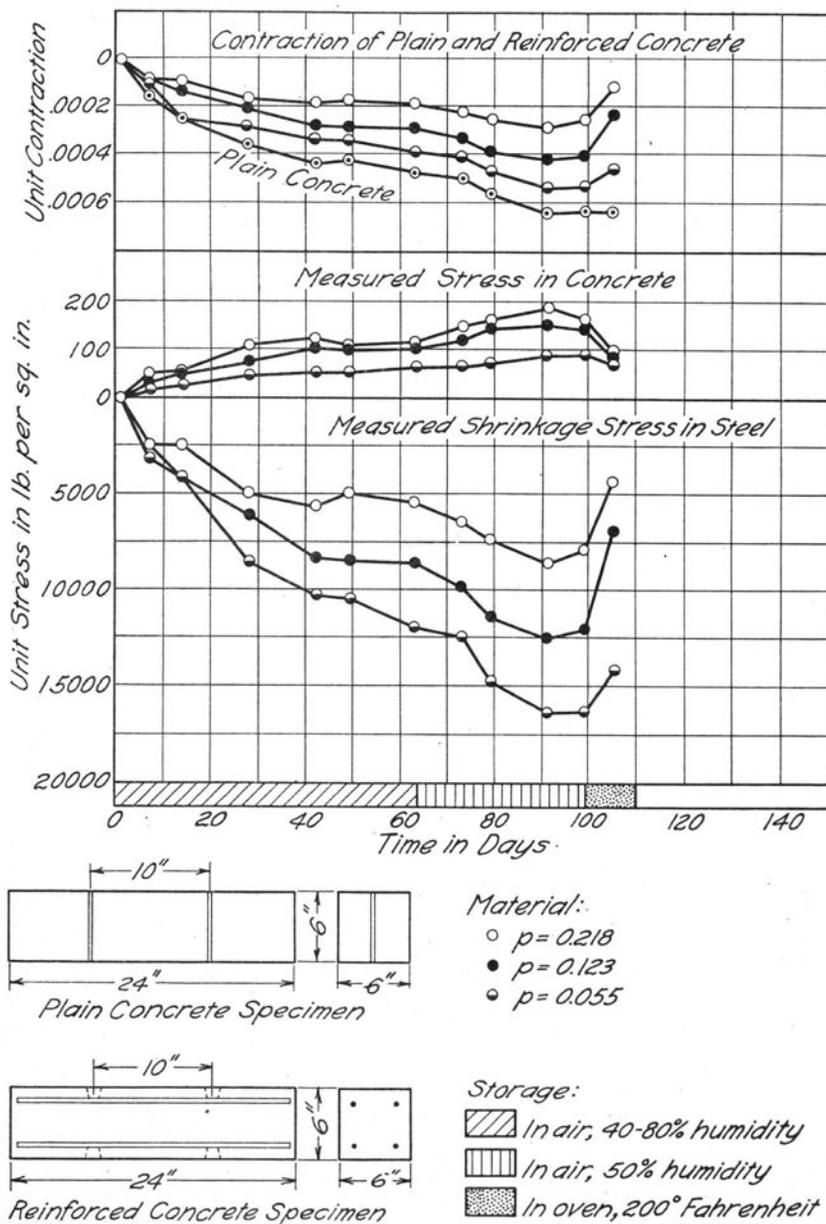


FIG. 10. SHRINKAGE STRESSES IN REINFORCED CONCRETE SPECIMENS

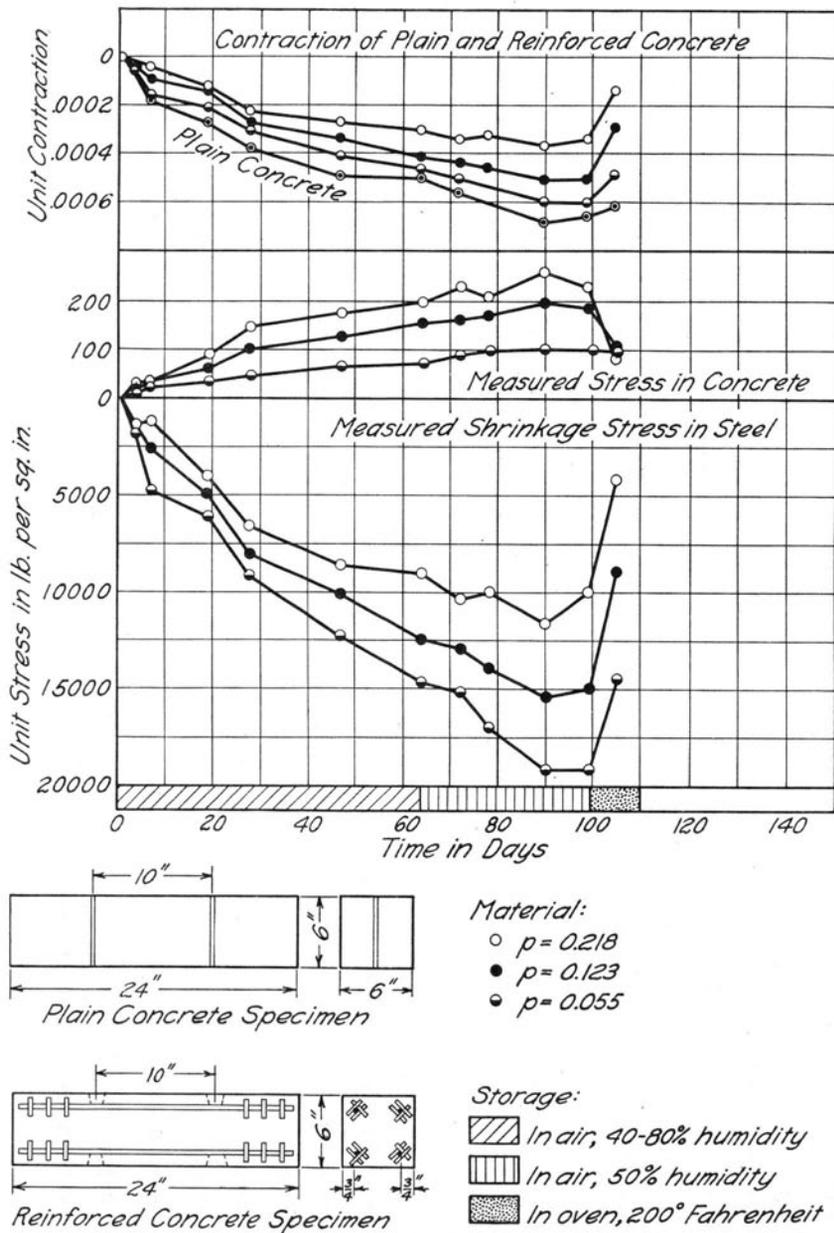


FIG. 11. SHRINKAGE STRESSES IN REINFORCED CONCRETE SPECIMENS

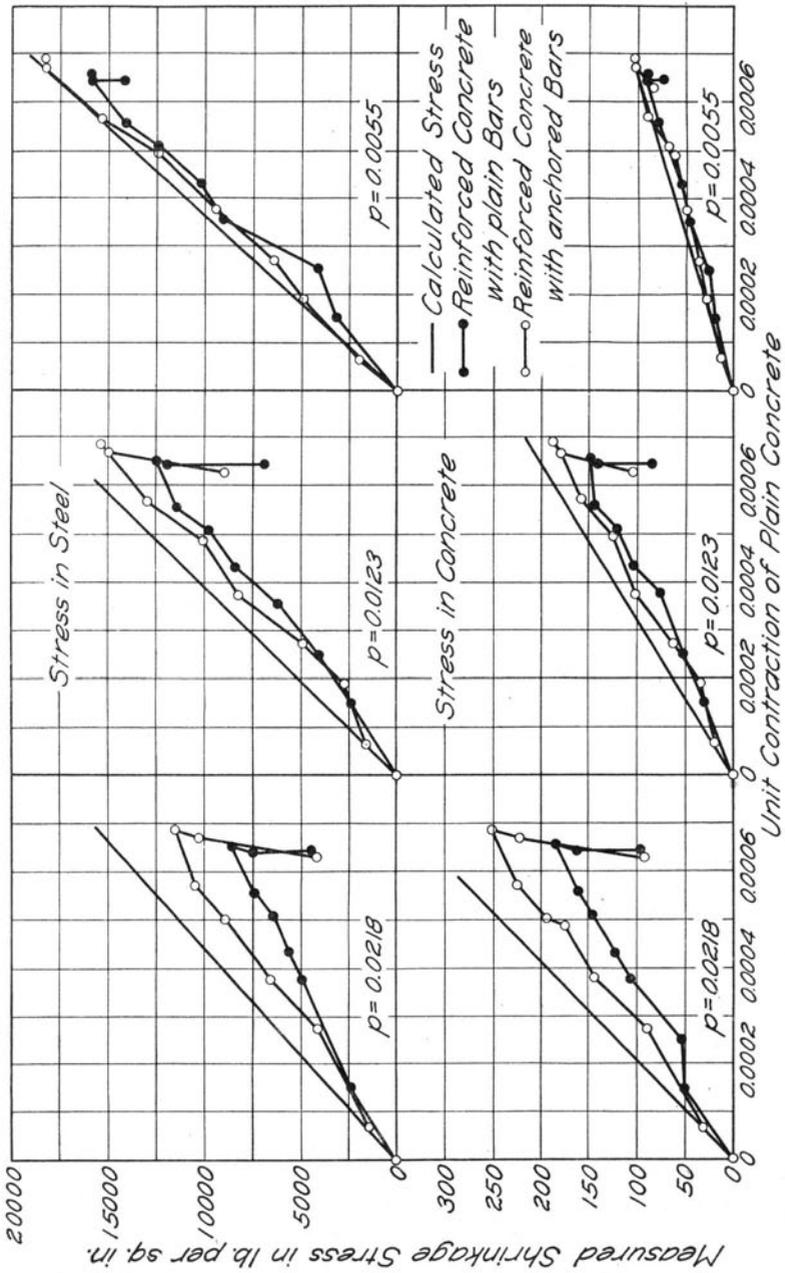


FIG. 12. COMPARISON OF MEASURED AND CALCULATED SHRINKAGE STRESSES

V. CONCLUSION

9. *Comments.*—The following comments are offered; part of them relate to the concrete used in the tests and part are inferences based on the writer's general experience and observation:

1. Concrete expands when it absorbs moisture and contracts when it is dried. Concrete of a 1:2:4 mixture is likely to contract during hardening as much as 0.05 per cent in an ordinary structure.

2. Contraction of concrete by the loss of moisture causes stress in the concrete when it is restrained by an external force. The amount of this stress is not as small as is generally supposed.

3. The shrinkage stress caused in the steel in reinforced concrete may reach the usually accepted working stress of steel when the amount of reinforcement is less than 1.5 per cent.

4. The shrinkage stress developed in 1:2:4 concrete may reach the ultimate tensile strength of the concrete when the amount of reinforcement is greater than 1.5 per cent. With richer mixtures the increase in shrinkage stress may be relatively greater than the increase in ultimate strength.

5. The greater the percentage of reinforcement the greater the tensile stress that may develop in the concrete, and concrete having a higher percentage of reinforcement than 1.5 per cent is likely to have cracks formed unless proper provision is made.

6. In reinforced concrete out of doors, subject to alternate wet and dry conditions, cracks may readily be formed under the repeated stress which is nearly equal to the tensile strength of the concrete.

7. Reinforced concrete does not appear likely to be a durable material in a place where a corrosive influence on steel, such as sea air, is active, unless proper protection against the formation of shrinkage cracks is made.

8. It is suggested that the prevention of shrinkage stress in concrete might be accomplished in two ways, either by finding a

cement giving less expansion and contraction, or by the use of a perfect waterproofing treatment.

9. It may be expected that an integral waterproofing compound might lessen the change of volume for a short time, but it would not prevent the final diffusion of moisture with consequent change in volume.

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